Large Symmetry Breaking and Mass Relations*

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The effects of symmetry breaking on the multiplet structure of composite particles are studied. The Sakata triplet is used for illustrations, and simple special models are studied where some mass regularities remain even in the presence of a fairly large symmetry breaking.

THE recent experimental discovery of the Ω^- hyperon[1] and the precise agreement of the observed baryon decuplet masses with the Gell-Mann–Okubo mass formula[2] based on unitary symmetry makes more acute the question why a result obtained by treating the symmetry breaking as small should work so well, since it is known that the unitary symmetry is badly broken. It seems useful, therefore, to study some special models where the symmetry is badly broken in the sense to be described below, and where some mass regularities nevertheless remain.

Specifically, we wish to consider composite systems made up of two or more particles, each of which belong to some irreducible representation of the basic symmetry group. Without symmetry breaking, the composite particles also belong to some irreducible representations. When the symmetry is broken, however, composite particles belonging to different irreducible representations may get mixed; or, worse still, the uniformity in interactions may be destroyed to the extent that some members of an irreducible representation cease to be bound (or cease to be resonances) while the other members still are. It is symmetry badly broken in this sense that we would like to study. Clearly in this case the assignment of observed approximate multiplets to irreducible representations of the group becomes highly ambiguous. The question is not so much one of group theory as one of dynamics. Unfortunately, dynamics is what we do not know how to treat properly. So in the following we will only consider an extremely crude and naive model, mainly to illustrate the breaking up of irreducible representations.

We will take as the basis of our model the unitary Sakata triplet, supplemented with some crude assumptions concerning the effects of symmetry breaking. If one does not wish to introduce more than three basic fields, to avoid either the use of highly unobservable fields or the possible emergence of groups larger than SU_3, the Sakata triplet is still the most appealing. On the other hand, the experimental occurrence of the reaction $P\bar{P}\rightarrow K_1K_2$ shows that the unitary Sakata symmetry is badly broken.[3] It is therefore ideal for our considerations. Let us consider the formation of bound states. When in suitable configurations there is supposed to be a strong attractive force between the Sakata triplets and anti-Sakata triplets of the order of some BeV per pair, causing the formation of composite systems, whereas the binding between Sakata triplets and Sakata triplets is supposed to be of the order of some MeV per pair. We consider the symmetry between Λ and N to manifest itself through the equality of the binding energies between systems which are identical in those respects such as the relative angular momentum and spin orientation of the constituent particles as well as their exchange symmetry properties, but differ in that a nucleon in one system is replaced by a Λ in the other.[4] Two such systems will have the same mass before the symmetry is broken. After the symmetry breaking the mass of the latter system will differ from the former system by $(M_\Lambda-M_N-\Delta)$, where Δ represents the *difference* in the binding energies of a Λ and a nucleon in the system. This holds for fermion systems. For boson systems it has been customary to take the $(\text{mass})^2$ as the proper variable for considering mass relations, although the precise reason for this is not yet completely understood. We will, nevertheless, also use the $(\text{mass})^2$ variable in the sense that for two boson systems which differ by the replacement of a nucleon by a Λ, the difference in the squares of the masses is given by $(M_\Lambda{}^2-M_N{}^2-\delta^2)$. In Appendix II we give a simple perturbation computation where such a $(\text{mass})^2$ dependence is natural, although the computation should not be taken seriously, and the origin at the $(\text{mass})^2$ dependence is best to be considered as still unknown.[5]

So far the discussion is fairly general. We now assume specifically that Δ and δ are approximately constant within a multiplet; i.e., when one replaces a nucleon by a Λ the *difference* in binding energy is Δ, and when one replaces a second nucleon by a Λ, the additional difference in binding energy is again Δ. Such an assumption is plausible if $(M_\Lambda-M_N)>\Delta$, and $(M_\Lambda{}^2-M_N{}^2)>\delta^2$, because then the changes in Δ are presumably even smaller than Δ so that $(M_\Lambda-M_N)\gg$(changes in Δ). One may ask, however, whether it is consistent to have

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[1] V. E. Barnes, P. L. Connolly, D. J. Crennell, B. B. Culwick, W. C. Delaney *et al.*, Phys. Rev. Letters 12, 204 (1964).

[2] M. Gell-Mann, California Institute of Technology, Synchrotron Report No. CTSL-20, 1961 (unpublished); S. Okubo, Progr. Theoret. Phys. (Kyoto) 27, 949 (1962).

[3] C. A. Levinson, H. J. Lipkin, S. Meshkov, A. Salam, and R. Munir, Phys. Letters 1, 125 (1962).

[4] The connection between such "substitution symmetry" and unitarity symmetry is discussed in Appendix I.

[5] See, however, Ref. 7.

$(M_\Lambda - M_N) > \Delta$, i.e., to have large mass shifts and small shifts in the binding energies, particularly since the binding energies themselves are of the order of some BeV per pair. For instance, if we consider the attraction between the baryons and antibaryons as arising from the exchange of particles, the range of interaction will be changed when the masses of the exchanged particles are shifted by symmetry breaking. The change in range will in turn affect the binding energies. However, it is instructive to recall in this respect some results from the analysis of Λ hypernuclei. It might seem at first that the Λ-nuclei binding should be much weaker than the corresponding nucleon-nuclei binding, both because K exchange is of a much shorter range (single pion exchange being forbidden for Λ), and because ΛKN coupling is relatively weak. The analysis by Dalitz and co-workers[6] found, however, that this is not so, that 2π exchange is probably more important so that the range is only $1/(2m_\pi)$ and not $1/m_K$, and that Λ binding is approximately of equal strength as the corresponding nucleon binding. Of course, one should not compare directly the Λ-nuclei binding with the presumed tighter binding of Λ to antibaryons. But it may be argued that the tighter the binding, the more important is the contribution from the exchange of many particles, which are relatively insensitive to the $\Lambda - N$ differences in quantum numbers. In any case, we will adopt as our working hypothesis that $(M_\Lambda - M_N) > \Delta$, and that the change in Δ within a multiplet is negligible. Recapitulating, we have within this model the "substitution rule," i.e., for fermion composite systems which are similar except for the replacement of a nucleon by a Λ, the mass difference between the two is given by $(M_\Lambda - M_N - \Delta)$; and for boson systems, the difference between the squares of the masses is given by $(M_\Lambda{}^2 - M_N{}^2 - \delta^2)$; Δ and δ are constant within a multiplet.

Having outlined the assumptions, we now examine the consequences.

A. BOSON SYSTEMS

Let us first consider boson systems made up of a baryon and an antibaryon. Out of the 3 Sakata triplets one obtains nine bosons. If one considers the mass splitting resulting from symmetry breaking by using the substitution rule above, omitting for the moment the singlet-octet splitting and the isotopic-spin splitting, one obtains the following 3 levels:

$$\Lambda\bar{\Lambda} \qquad M_3,$$

$$\Lambda\bar{n},\ \Lambda\bar{p},\ p\bar{\Lambda},\ n\bar{\Lambda} \qquad M_2,$$

$$(1/\sqrt{2})(p\bar{p}+n\bar{n});\ n\bar{p},\ p\bar{p},\ (1/\sqrt{2})(p\bar{p}-n\bar{n}) \quad M_1.$$

On the other hand, if one considers the singlet-octet splitting resulting from symmetry-preserving interaction, omitting for the moment the symmetry breaking effects, one gets two levels,

$$(1/\sqrt{3})(p\bar{p}+n\bar{n}+\Lambda\bar{\Lambda}) \qquad m_s,$$

$$\Lambda\bar{p},\ \Lambda\bar{n},\ p\bar{\Lambda},\ n\bar{\Lambda},\ (1/\sqrt{2})(p\bar{p}-n\bar{n}),$$
$$p\bar{n},\ n\bar{p},\ (1/6^{1/2})(p\bar{p}+n\bar{n}-2\Lambda\bar{\Lambda}) \quad m_o.$$

The problem of which classification should be used as a first approximation is in this case an unambiguous and quantitative one; one simply compares $(M_3-M_2) = (M_2-M_1)$ with (m_s-m_o). If (M_2-M_1) is larger, the first classification should be used. On the other hand, if (m_s-m_o) is much larger, the second classification should be used, with the symmetry breaking effects put in as a further correction. Now $(M_2{}^2-M_1{}^2)$ can be estimated in this model to be of the order of $\Lambda - N$ (mass)^2 difference, but (m_s-m_o) cannot be estimated from a general knowledge of the strength of the symmetry-preserving interaction alone, because the singlet-octet splitting may depend very much on the particular configuration of the composite systems. We will therefore have to look at the physical spectra.

First let us consider the vector bosons. Before one decides which classification to use, one can at least assign $(K^*)^+$ to $\bar{\Lambda}p$, and ρ^+ to $\bar{n}p$, this assignment being the same for the two different classifications. This gives $(M_2-M_1)=138$ MeV. From the masses of K^* and ρ and using the Okubo formula one can also find m_o (i.e., m_{octet} in the absence of symmetry breaking) to be $m_o = 845$ MeV. On the other hand, m_s should not be too far from the mass of ω(728 MeV), if ω is considered as the singlet. Thus $(M_2-M_1) > |m_\omega - m_{\text{octet}}| \gtrsim |m_s - m_o|$. So in this case, the first classification should be used, and one assigns[7]

$$\rho = [n\bar{p}, (1/\sqrt{2})(p\bar{p}-n\bar{n}), p\bar{n}],$$
$$\omega = (1/\sqrt{2})(p\bar{p}+n\bar{n}),$$
$$K^* = (\Lambda\bar{n}, \Lambda\bar{p}, p\bar{\Lambda}, n\bar{\Lambda}),$$
$$\phi = \Lambda\bar{\Lambda}.$$

The "substitution rule" gives the mass relations

$$m_\rho{}^2 = m_\omega{}^2, \tag{1}$$

$$m_{k^*}{}^2 - m_\rho{}^2 = m_\phi{}^2 - m_{k^*}{}^2 = (M_\Lambda{}^2 - M_N{}^2 - \delta^2). \tag{2}$$

Equation (2) with m_{k^*} and m_ρ as inputs gives $m_\phi = 1010$ MeV, as compared with the observed 1020 MeV. Also $\delta^2 = 0.226\ \text{BeV}^2 < (M_\Lambda{}^2 - M_N{}^2)$.

We next turn to the pseudoscalar particles. Here by similar considerations as above one finds $(M_2-M_1) = 356$ MeV, but experimentally one does not see a particule corresponding to the singlet. Hence $(m_s-m_o) > (2M_N - m_o) \gg (M_2-M_1)$. By our criterion it is more appropriate to use the second classification. Thus

$$\pi = [p\bar{n}, 1/2(p\bar{p}-n\bar{n}), n\bar{p}],$$
$$K = [\Lambda\bar{n}, \Lambda\bar{p}, p\bar{\Lambda}, n\bar{\Lambda}],$$
$$\eta = (1/6^{1/2})(p\bar{p}+n\bar{n}-2\Lambda\bar{\Lambda}).$$

[6] R. H. Dalitz and B. W. Downs, Phys. Rev. 111, 967 (1958).

[7] While this work is in progress we receive a report by F. Gursey, T. D. Lee, and M. Nauenberg (to be published) where a similar assignment is made.

From the substitution rule for mass differences one gets

$$m_k^2-m_\pi^2=(M_\Lambda^2-M_N^2-\delta'^2)\,, \tag{3}$$

$$4m_k^2=3m_\eta^2+m_\pi^2\,. \tag{4}$$

Equation (4) is, of course, just the Gell-Mann–Okubo formula; but Eq. (3) gives $\delta'^2=0.226\ \mathrm{BeV}^2=\delta^2$; i.e.,

$$m_k^2-m_\pi^2=m_{k^*}^2-m_\rho^2\,. \tag{5}$$

The amazing accuracy with which Eq. (5)[8] is satisfied by the observed masses of the particles seems to indicate that δ is not only constant within a multiplet, but is also the same for some different multiplets.[9]

B. FERMION SYSTEMS

Next, we examine fermion systems. As is well known, the Sakata model does not give either octets or decouplets with baryon number one. Nevertheless, let us consider the smallest number of particles that can make a strangeness minus three hpyeron; it must clearly be composed of 2 antibaryons and 3 baryons. Suppose we consider the 2 antibaryons as forming a core, and require it to be stable as a two-particle system. The only known stable system is that of the $T=0$, $J=1$ state $(\bar{p}\bar{n}-\bar{n}\bar{p})$. The remaining 3 baryons taken in the completely symmetric state can now combine with the "core" to form a baryon number one decuplet[10]:

$$(\bar{p}\bar{n}-\bar{n}\bar{p})NNN \quad \text{in} \quad T=\tfrac{3}{2}\ \text{state}\,,$$
$$(\bar{p}\bar{n}-\bar{n}\bar{p})NN\Lambda \quad \text{in} \quad T=1\ \text{state}\,,$$
$$(\bar{p}\bar{n}-\bar{n}\bar{p})N\Lambda\Lambda \qquad T=\tfrac{1}{2}\,,$$
$$(\bar{p}\bar{n}-\bar{n}\bar{p})\Lambda\Lambda\Lambda \qquad T=0\,.$$

This fits the observed decuplet nicely. From the "substitution rule" one gets the equal spacing rule for mass splitting, with $\Delta=29$ MeV $\ll(M_\Lambda-M_N)$, so that the assumption of the constancy of Δ should be particularly good for this multiplet.

The fact that for boson systems δ not only remains constant within a multiplet, but is also the same for different multiplets as indicated by Eq. (5), leads one to examine whether Δ can also be approximately the same for different multiplets. For example, if one considers the Y_1^* at 1660 MeV[11] to be obtainable from the N^* at 1512 MeV by the substitution rule, one obtains $\Delta'=30$ MeV, i.e.,

$$Y_1^*(1660)-N^*(1512)=Y_1^*(1385)-N^*(1238)\,, \tag{6}$$

accurate to within 1 MeV. If the particles on the left-hand side of Eq. (6) are also formed from 5 Sakatons, there should be a Ξ^{**} at 1808 MeV, and an Ω^{-*} at 1956 MeV.

Finally, by the same method one can also construct a baryon number one octet satisfying the Gell-Mann–Okubo formula. However, having singled out N and Λ as the fundamental particles there is not more reason to expect that N, Λ, Σ, and Ξ should form a multiplet, and this possibility will not be exploited.

Needless to say, the assumption of the particular mechanism of binding requiring a stable core is purely *ad hoc.* We are not arguing for the plausibility of this particular mechanism, but rather for the possibility, which cannot be excluded as long as we are ignorant of the dynamics, that such things may happen, altering the connection between multiplets and irreducible representations very drastically, while still leaving some mass regularities which are more or less precise. The model considered is completely trivial, its main virtue being that no unphysical fields or particles need to be introduced. We used it to emphasize that whether a particular symmetry breaking is to be considered large or small is probably a very complicated question: It depends on what the symmetry breaking effect is compared to. In the examples above we have tried to illustrate this twice. In the baryon example symmetry-breaking effects of a few MeV are very small compared to the baryon-antibaryon binding, but are large for the baryon-baryon "core"; (after all, it takes only a few MeV's difference in binding energies to make the deuteron bound and the corresponding Λ-N system most likely unbound). In the meson example the symmetry-breaking effects of a few hundred MeV (Λ-N mass difference) are small when compared with a large singlet-octet splitting as was assumed for the pseudoscalar mesons, but are large when the singlet and the octet happen to lie close together as was assumed for the vector mesons in the example. In either the fermion or the boson case, the symmetry breaking effect is large in some respect, but are small compared to the primary baryon-antibaryon binding of the order of BeV's per pair. When one can separate out the "large" effects (which in these examples consist of changing the multiplet structures), then it is not necessarily inconsistent that the remaining "small" effects give rise to fairly precise mass regularities.

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APPENDIX I

There has been some lack of precision in the literature in speaking of the connection between unitary symmetries and the invariance of the interaction under sub-

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[8] This relation has also been noted by Schwinger: Phys. Rev. Letters **12**, 237 (1964).

[9] See also Eq. (6) in the following.

[10] The three-baryon states with different symmetry properties can of course have very different interactions with the "core."

[11] L. W. Alvarez, M. H. Alston, M. Ferro-Luzzi, D. O. Huwe, G. R. Kalbfleisch, *et al.*, Phys. Rev. Letters **10**, 184 (1963); P. L. Bastien and J. P. Berge, *ibid.* **10**, 188 (1963); and Ref. 9. M. Taher-Zadeh, D. J. Prowse, P. E. Schlein, W. E. Slatar, D. H. Stork, *et al.*, *ibid.* **11**, 470 (1963). This relation Eq. (6) will be just a coincidence if the parity of $Y_1^*(1660)$ turns out to be different from that of $N^*(1512)$. The last reference gives some indication of positive parity, but the evidence is not yet conclusive.

stitution of one particle for another. Sometimes it is loosely said that unitary symmetry would imply the substitution invariance; on the other hand, there are attempts to "derive" the unitary symmetry from the invariance of the interaction under substitution.[12] For instance, in the Nagoya model[12] the Sakata triplet is supposed to be obtained by attaching a B matter to the leptons e, μ, and ν. Only the B matter is supposed to be responsible for strong interactions; e, μ, and ν then serve only as labels, and the interaction is invariant under "substitutions." From this it was argued that one could obtain unitary symmetry. To examine this problem more closely it is useful to consider the simpler case of 2-dimensional unitary symmetry, using p and n as our basic objects. First of all, one should define what substitution invariance really means. It could either mean, using $V(AB)$ to denote $(AB|H|AB)$,

(a) $V(pn)=V(pp)=V(nn)$; $V(\bar{p}n)=V(\bar{n}p)=V(\bar{p}p)=V(\bar{n}n)$, as long as the two particles are in a given spin and angular momentum state; or mean

(b) $V(pp)=V(nn)=V[(1/\sqrt{2})(pn+np)]$,
$V(\bar{p}n)=V(\bar{n}p)$;
$V(p\bar{p})=V(n\bar{n})$
with the same conditions as in (a).

It is easy to see that the statement (a) is stronger than 2-dimensional unitary symmetry, and the conditions (b) are weaker than and implied by the unitary symmetry. The first line in (b) is identical to (a) because of the Pauli principle. But the fact that $p\bar{p}$ can transform into $n\bar{n}$, and the consequent symmetriaztion and antisymmetrization with respect to p-n interchange does not provide any relations between the second and the third line in (b). This is because the state $(1/\sqrt{2})\times(p\bar{p}+n\bar{n})$ has just as much right to be assigned together with the $\bar{n}p$ and $\bar{p}n$ states on exchange symmetry grounds as the $(1/\sqrt{2})(p\bar{p}-n\bar{n})$ state. In other words, the transformation $p\leftrightarrow\bar{n}$, $n\leftrightarrow\bar{p}$ has as much meaning *a priori* as the transformation $p\leftrightarrow-\bar{n}$, $n\leftrightarrow\bar{p}$, since both will take the states $\bar{p}n$ and $\bar{n}p$ into themselves. The state $(1/\sqrt{2})(p\bar{p}-n\bar{n})$ gets singled out to be associated together with $\bar{p}n$ and $\bar{n}p$ only when one invokes G parity as physically meaningful, but the use of G parity presumes, of course, isotopic spin invariance. Thus, "substitution invariance" in the sense of (b) is weaker than the unitary symmetry, and it is this weaker substitution invariance that we have used in this paper.

APPENDIX II

In this Appendix we assume ρ to be a bound state of $N\bar{N}$ and ϕ to be a bound state of $\Lambda\bar{\Lambda}$, and compute the contribution to the ρ mass-renormalization from the $N\bar{N}$ loop alone, and the contribution to the ϕ mass-renormalization from the $\Lambda\bar{\Lambda}$ loop alone, taking the couplings $g_{\rho N\bar{N}}$ and $g_{\phi\Lambda\bar{\Lambda}}$ to be equal, but with $M_N\neq M_\Lambda$. The integrals from the loop contributions are quadratically divergent, but their difference is convergent. This fact was pointed out to the author by Cornwall. One finds, to first order in the mass differences

$$m_\phi^2-m_\rho^2=(M_\Lambda^2-M_N^2)\times[(\partial/\partial M^2)\Pi(M,M,K^2)]_{M^2=M_\Lambda^2,\,K^2=m_\phi^2},\quad\text{(A1)}$$

where

$$\left[\frac{\partial}{\partial M^2}\Pi(M,M,K^2)\right]_{M^2=M_\Lambda^2,K^2=m_\phi^2}=\left(\frac{g^2}{4\pi}\right)\frac{1}{2\pi}\left\{\frac{16M_\Lambda^2}{m_\phi(4M_\Lambda^2-m_\phi^2)^{1/2}}\times\tan^{-1}\left[\frac{m_\phi}{(4M_\Lambda^2-m_\phi^2)^{1/2}}\right]^{-3}\right\}.\quad\text{(A2)}$$

Likewise, if one assumes K^* to be a bound state of $N\bar{\Lambda}$ and compute the $N\bar{\Lambda}$ loop, one finds

$$m_{K^*}^2-m_\rho^2=(M_\Lambda-M_N)[(\partial/\partial M_1)\times\Pi(M_1,M_2,K^2)]_{M_1=M_2=M_\Lambda,K^2=m_{K^*}^2}\quad\text{(A3)}$$

or

$$M_{K^*}^2-m_\rho^2=\tfrac{1}{2}(M_\Lambda^2-M_N^2)[(\partial/\partial M^2)\times\Pi(M,M,K^2)]_{M^2=M_\Lambda^2,K^2=m_{K^*}^2}.\quad\text{(A4)}$$

Because the derivative of $\Pi(M_1,M_2,K^2)$ is not sensitive to the values of M and K^2, i.e.,

$$[(\partial/\partial M^2)\Pi(M,M,K^2)]_{M^2=M_\Lambda^2.K^2=m_\phi^2}\approx[(\partial/\partial M^2)\Pi(M,M,K^2)]_{M^2=M_N^2.K^2=m_\rho^2},$$

we have evaluated it at $M^2=M_\Lambda^2$ in Eqs. (A1) to (A4). From this it also follows that

$$m_\phi^2-m_{K^*}^2\approx m_{K^*}^2-m_\rho^2.\quad\text{(A5)}$$

If we substitute the values of the physical masses in Eq. (A1) and (A2), we find $(g^2/4\pi)\approx5$, as compared to the known $\rho N\bar{N}$ coupling of roughly $(g_{\rho N\bar{N}}^2/4\pi)\approx1$.

This computation is not to be taken seriously since it is a perturbation computation and since the assumption of $g_{\rho N\bar{N}}=g_{\phi\Lambda\bar{\Lambda}}$ is really unphysical. It serves only to illustrate how the mass-squared dependence is natural in such a computation, that the relation (A5) obtains, and that Eq. (A1) relating the mass differences to selected self-energy contributions is numerically of the right order of magnitude.

[*Note added in proof.* After this paper went to print, evidence for a Ξ^{**} at 1810 MeV was reported by G. A. Smith, J. S. Lindsey, J. J. Murray, J. Shafer, A. Galtieri, O. I. Dahl, P. Eberhard, W. H. Humphrey, G. R. Kalbfleisch, D. R. Ross, F. T. Shively, R. D. Tripp, Phys. Rev. Letters **13**, 61 (1964). The mass is in remarkable agreement with the predicted value of 1808 MeV.]

[12] Z. Maki, M. Nakagawa, Y. Ohnuki, and S. Sakata, Progr. Theoret. Phys. (Kyoto) 23, 1174 (1960). See also E. Abers, F. Zachariasen, and C. Zemach, Phys. Rev. 132, 1831 (1963).